corrigé EM
L2 P-SPC mars 2012

(I) MF. $\vec{Rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$; $\oint_{(c)} \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot\vec{n}\,dS$

MA $\vec{Rot}\,\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0 \frac{\partial\vec{E}}{\partial t}$

$\oint_{(c)}\vec{B}\cdot\vec{dl} = \mu_0(I+I_d)$ $\quad I=\iint_S \vec{J}\cdot\vec{n}\,dS$

$I_d=\varepsilon_0\iint \frac{\partial \vec{E}}{\partial t}\cdot\vec{n}\,dS$

(II) 1) $\phi_0 = \iint_S \vec{B}_0\cdot\vec{n}\,dS = \mu_0 n I \iint_S \vec{e}_z\cdot\vec{e}_z\,dS$

$n=\frac{N}{\ell}$

$\phi_0=\mu_0 n I\pi R^2$

$\phi_p = N(\mu_0 n I\pi R^2) = \frac{N^2\mu_0 I\pi R^2}{\ell}$

2) $\phi_p = LI = \mu_0\frac{N^2}{\ell}\pi R^2 I$

$L=\mu_0\frac{N^2}{\ell}\pi R^2 = \frac{\mu_0 n^2\ell^2\pi R^2}{\ell} = \mu_0 n^2\ell\pi R^2$

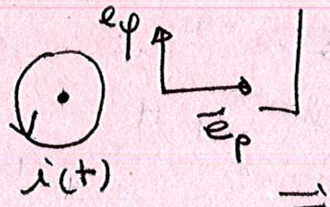
3) $\mathcal{E}_m = \iiint_V \frac{B_0^2}{2\mu_0}\,dV = \frac{\mu_0^2 n^2 I^2}{2\mu_0}\iiint 2\pi\rho\,d\rho\,dz$

$= \mu_0 n^2 I^2\pi\int_0^R \rho\,d\rho\int_0^\ell dz = \mu_0 n^2 I^2\pi\,\frac{1}{2}R^2\ell$

$=\frac{1}{2}(\mu_0 n^2\pi R^2\ell)\,I^2 = \frac{1}{2}LI^2$

$L=\mu_0 n^2\pi R^2\ell$

4)



$(\vec{e}_\rho,\vec{e}_\varphi) \equiv$ plan de symétrie

$(\vec{e}_\rho,\vec{e}_z) \equiv$ plan d'antisymétrie

$\vec{E} \perp (\vec{e}_\rho,\vec{e}_z) \Rightarrow \vec{E} \parallel \vec{e}_\varphi$

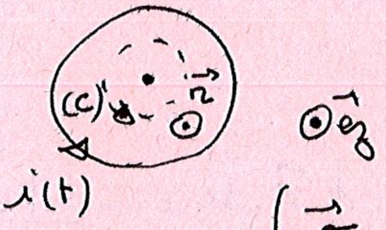
$(\ell \gg R) \Rightarrow$ invariance / z $\rightarrow$ symétrie de révolution / $z'z$

$\Rightarrow$ invariance / φ

$$\vec{E}_1 = E_{1\varphi}(\rho)\,\vec{e}_\varphi$$

5)

$$\vec{n} \equiv \vec{e}_z$$



$$\oint_C \vec{E}_1 \cdot \vec{dl} = -\frac{d\Phi_{B_0}(t)}{dt}$$

$$\oint_C E_{1\varphi}\,\vec{e}_\varphi\;\rho\, d\varphi\,\vec{e}_\varphi = -\frac{d}{dt}\iint_S \vec{B}_0(t)\,\vec{n}\,dS$$

$$E_{1\varphi}\,2\pi\rho = -\frac{d}{dt}\iint_S B_{0m}\,\cancel{\sin}\,\omega t\;\vec{e}_z\cdot\vec{e}_z\,dS$$

$$E_{1\varphi}\,2\cancel{\pi}\rho = -\frac{d}{dt}(\cos\omega t)\,B_{0m}\,\cancel{\pi}\rho^2$$

$$\boxed{E_{1\varphi} = \frac{B_{0m}\,\omega\,\sin\omega t\,\rho}{2}}$$

III)

$$\Phi_{21} = \iint_{S_2} \vec{B}_1 \cdot \vec{n}_2\, dS_2 \qquad \vec{n}_2 = \vec{e}_\varphi$$

$$= \frac{\mu_0 I_1}{2\pi}\iint_{S_2} \vec{e}_\varphi\cdot\vec{e}_\varphi\;\frac{d\rho}{\rho}\,dz$$

$$= \frac{\mu_0 I_1}{2\pi}\, b\int_a^{a+b}\frac{d\rho}{\rho} = \frac{\mu_0 I_1}{2\pi}\, b\ln\left(\frac{b+a}{a}\right)$$

$$\Phi_T = N_2\,\Phi_{12} = M I_1$$

2)

$$M = N_2\frac{\mu_0}{2\pi}\, b\ln\left(\frac{a+b}{a}\right) = \frac{\cancel{4\pi}10^{-7}}{\cancel{2\pi}}\,\cancel{\times 2\,10^{-2}}\ln 3 \times 2\,10^{3}$$

$$= 8\,10^{-6}\ln 3 \simeq$$

$$\simeq 9\,10^{-6}\ \mu H$$

3)

$$e_2 = -\frac{d\Phi_T}{dt} = -M\frac{dI_1}{dt} = +M I_{1m}\,\omega\,\sin\omega t$$

(1,4 A)

$$M\frac{I_{1m}}{\sqrt{2}}\,\omega = \Gamma_A\frac{I_{2M}}{\sqrt{2}} \qquad I_{1\,eff} = \frac{\Gamma_A}{M\omega}\,I_{2\,eff} = \frac{4}{9\,10^{-6}\,2\pi\,50} \simeq$$